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A Transportation Location-Allocation Model for Regional Blood Banking

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A Transportation Location-Allocation Model for Regional Blood Banking

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Abstract: In recent years, there has been much discussion about the issue of regionalization of blood banking systems. In this work we focus on the transportation location-allocation aspects of regionalization. We are given the locations and expected blood requirements of a set of N hospitals. Each hospital is to be assigned to a regional blood bank which will periodically supply the hospital's expected blood requirement for the period, as well as supply its emergency blood demands at the time of the emergency. The blood shipments are to be made by special delivery vehicles which have given capacities and given limits on the number of deliveries they can make per day. We present algorithms to decide how many blood banks to set up, where to locate them, how to allocate the hospitals to the banks, and how to route the periodic supply operation, so that the total of transportation costs (periodic and emergency supply costs) and the system costs are minimum. The algorithms are tested on data from the Chicago area where very good results are obtained.

■ Blood banks are an important and integral part of health service systems. Their main functions are blood procurement, processing, cross-matching, storage, distribution, recycling, pricing, quality control and outdating. The large blood banks are often also responsible for blood research, disease and reaction prevention. In recent years, there has been much discussion on the issue of regionalization of blood banking systems, in the hope of decreasing shortages, outdates and operating costs, without sacrificing blood quality, research and education.

In a broad sense, regionalization is a process by which blood banks within a given geographical area move toward the coordination of their activities. Such coordination may range from cases in which the blood banks merge into a large, centralized unit, to cases where the existing structure remains unaltered and only certain functions, such as donor recruitment, processing and distribution, are coordinated among the blood banks. In most of these cases questions of optimal region size, central and local bank locations, regional boundaries, optimal distribution and communication network configurations must be answered. Also, administrative policies, ordering and crossmatching policies, and donor recruitment and component therapy strategies must be analyzed and coordinated. (See Cohen et al. [11] for a detailed discussion of the advantages, disadvantages, benefits and costs of regionalization.)

In this paper we focus on the location-allocation-transportation aspects of regionalization. Our variables will be central bank locations, regional boundaries and blood distribution network configurations. In the model below, all the other aspects of regionalization are summarized by the terms, system costs, which are primarily functions of two factors: the number of hospitals in a region and the amount of blood used by each hospital in the region. Both of these factors are functionally related to the variables considered in the location-allocation-transportation model (since they vary with the regional boundaries). The other factors that affect the nontransportation aspects of regionalization tend to be independent of the variables in the model, consequently they are independent of the location-allocation-transportation decisions.

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The regionalization model is verbally described as follows: "Within a given geographical area, there are N hospitals. Regionalization is to be achieved by dividing the area into M regions and establishing a central blood bank in each region. All blood banking activities in a region are to be coordinated. Supply generation is to be done mainly by each central bank and each hospital is to obtain its primary blood supply from the central bank in its region. The blood distribution operation consists of periodic and emergency deliveries. The hospitals in a region receive their periodic daily requirements from their central bank. The blood deliveries are made by vehicles which, starting from the central bank, visit one by one the hospitals they are scheduled to supply, and return to the central bank. These vehicles have given capacities and given limits on the number of deliveries they can make per day. Because of the wide fluctuations in demand, a hospital may deplete certain blood types before the next periodic delivery is due. In that case, a delivery vehicle is dispatched immediately, from its central bank. The delivery vehicle makes an emergency blood delivery to that hospital and returns to the blood bank. The problem is to decide how many central blood banks to set up, where to locate them, how to allocate the hospitals to the banks, and how to route the periodic supply operation, so that the total of transportation costs (periodic and emergency supply costs) and the other system costs are a minimum." This problem will be called the Blood Transportation-Allocation Problem (BTAP).

In modeling the BTAP it is necessary to describe the costs as functions of the decision variables. The periodic delivery costs, which are a set of linear terms, depend on all three types of variables in this problem, that is, routing the periodic supply operations, locating the banks, and allocating the hospitals. The emergency costs are also a set of linear terms and depend on only two types of variables, locating the banks and allocating the hospitals. The system costs are nonlinear functions of the size of the blood banks and the number of hospitals allocated to the blood banks, and therefore, of all the variables in this problem they depend only on hospital allocations. So, if the system costs were constant and the emergency costs were negligible, the model would be equivalent to the General Transportation Problem (GTP) (see Or, [33] or Magnanti et al., [32]). If the system costs were constant and the periodic delivery costs were negligible, the model would reduce to a Location-Allocation problem (LAP) (see Cooper [12, 13], Hurter and Wendell [42, 43], or Francis and White [15]. So the model is a complex combination of these two large problems. The basic strategy we use in order to obtain a good solution depends heavily on these two subproblems. We solve each subproblem independently and then combine them at the end, making tradeoffs between them and superimposing the system costs considerations, to obtain a good solution to the model. However, it should be noted that, unlike the GTP, solving the LAP does not produce a complete, feasible solution to the main model. It only gives the locations and the allocations, and in order to get the missing periodic delivery routes, one must solve a set of vehicle dispatch problems.

Other work on regionalization or centralization of blood bank activities has not looked at the location-allocation-transportation aspects. Jennings [23, 24] used a simulation model to construct part of a regional blood banking system. He grouped a number of identical hospitals together; however, he did not have a central blood bank. Transshipment policies and inventory levels were studied to see their impact on shortages and outdates. Yen [46] also studied multi-echelon inventory systems. He concentrated his efforts on the optimal inventory levels and the optimal issuing policies. Prastacos [38] and Prastacos et al. [37] were interested in the allocation of existing stocks among the hospitals. Neither Jennings, Yen nor Prastacos et al. studied the location of central banks or the allocation of hospitals to them.

The Blood Transportation-Allocation Model

Because the BTAP is a complex optimization problem, we will make a few reasonable assumptions to decompose the problem into smaller subproblems.

ASSUMPTION 1: The number of banks, M, is a given constant number.

Even in cases in which the above assumption does not hold, M is almost always restricted to a small, finite, feasible set (M is always an integer and $1 \leqslant M \leqslant N$). So, in those cases one could solve the problem for each feasible value of M to get the optimal solution. In this respect, assumption 1 is not restrictive.

ASSUMPTION 2: The blood delivery period is daily for each hospital and is an input parameter to the BTAP.

Considering that some hospitals use more than 7000 units of blood per year, while some others use less than 10, this is an unrealistic assumption. Unfortuantely, determining the optimal multiple delivery periods as well as the location–allocation and routings increase the complexity of the problem considerably and make it almost impossible to find a direct solution procedure. A simple, multiple period problem should have three options for the periodic deliveries (daily, biweekly, weekly). However, the problem of choosing optimal periods for each hospital would be very large (and time consuming) combinatoric process. If the periods are set in advance (daily, biweekly, weekly) these multiple periods can be easily incorporated into the present location–allocation model merely by adjusting the costs to reflect costs per day. The routes of the delivery vehicles however would have to be adjusted later.

ASSUMPTION 3: The potential locations of the M banks are given.

Even in cases where this assumption does not hold, the set of feasible locations is almost always a small, finite set (usually one does not want to build a blood bank from scratch; instead, they are most frequently located in the area's largest hospitals or at existing blood centers). So, if necessary, we could solve the problem for all combinations of feasible locations, to get the optimal solution. In a design problem, this is not an impossible enumeration, since even a region the size of the Chicago metropolitan area (using the amount of blood transfused as a measure of hospital size) has only six central blood banks and has only seven hospitals with consumption rates of over 7000 units per year. Most of the latter do not qualify to be central blood banks for various reasons. So, in this respect Assumption 3 is not very restrictive.

The following notation will be used in formulating the BTAP.

i) N is the number of demand points.

ii) M is the number of supply points.

iii) n is the maximum number of supply vehicles available.

iv) $D = \{H_1, \ldots, H_N\}$ is a set of N demand points.

v) $S = \{H_{N+1}, \ldots, H_{N+M}\}$ is a set of M supply points.

vi) $H = D \cup S$ is the set of all points involved in the problem.

vii) d_{ij} is the "distance" from H_i to H_j. It should be noted that although Euclidean distances among locations of hospitals and central banks are used in the solution procedure, one could obtain a matrix of accurate travel times between all pairs of hospitals and banks, and one could use this matrix or any other "distance measure" instead of the Euclidean distance matrix.

viii) C_k, $k = 1, \ldots, n$ is the capacity of supply vehicle k.

ix) Q_i, $i = 1, \ldots, N$ is the requirement of demand point i.

x) D_k, $k = 1, \ldots, n$ is the maximum distance supply vehicle k may travel.

xi) γ_i, $i = 1, \ldots, N$ is the expected number of emergency deliveries to hospital H_i per period. γ_i is the probability that the demand at H_i exceeds the supply at H_i given the optimal inventory level at H_i is used.

xii) $s(\ell, k)$ is the systems cost function of a region, where ℓ is the number of hospitals in that region, and k is the amount of blood used per year in that region.

xiii) y_{ij}, $i = 1, \ldots, N; j = N + 1, \ldots, N+M$ is a zero-one variable such that y_{ij} is 1 if hospital H_i is assigned to central bank H_j and is 0 otherwise.

xiv) x_{ijk}, $i = 1, \ldots, N+M; j = 1, \ldots, N+M; k = 1, \ldots, n$ is a zero-one variable such that x_{ijk} is 1 if vehicle k goes from hospital H_i to H_j and is 0 otherwise.

The BTAP is:

Problem 1

$$\min\ z^1(x,y) = \sum_{i=1}^{N+M} \sum_{j=1}^{N+M} \sum_{k=1}^{n} d_{ij} x_{ijk} + \sum_{i=1}^{N} \sum_{j=N+1}^{N+M} \gamma_i d_{ij} y_{ij}$$

$$+ \sum_{j=N+1}^{N+M} s\left[\sum_{i=1}^{N} y_{ij}, \sum_{i=1}^{N} (300 Q_i)(y_{ij})\right] \quad (1)$$

subject to

$$\sum_{k=1}^{n} \sum_{j=1}^{N+M} x_{ijk} = 1 \quad i = 1, \ldots, N \quad (2)$$

$$\sum_{j=1}^{N+M} \sum_{i=1}^{N} Q_i x_{ijk} \leqslant C_k \quad k = 1, \ldots, n \quad (3)$$

$$\sum_{j=1}^{N+M} \sum_{i=1}^{N+M} d_{ij} x_{ijk} \leqslant D_k \quad k = 1, \ldots, n \quad (4)$$

$$\sum_{\{i: H_i \in S\}} \sum_{\{j: H_j \in \bar{S}\}} \sum_{k=1}^{n} x_{ijk} \geqslant 1 \quad \text{for all } (S, \bar{S}) \quad (5)$$

where S is any proper subset of H containing S and $\bar{S}$ is the complement of S.

$$\sum_{j=1}^{N+M} x_{hjk} = \sum_{i=1}^{N+M} x_{ihk} \quad k = 1, \ldots, n; h = 1, \ldots, N+M \quad (6)$$

$$y_{ij} \geqslant \sum_{h=1}^{N+M} x_{ihk} + \sum_{h=1}^{N+M} x_{jhk} - 1 \quad i = 1, \ldots, N;\ j = N+1, \ldots, N+M$$

$$k = 1, \ldots, n \quad (7)$$

$$x_{ijk} = 0, 1 \quad i = 1, \ldots, N+M;\ j = 1, \ldots, N+M;\ k = 1, \ldots, n \quad (8)$$

(note that $x_{iik} = 0$)

$$y_{ij} = 0, 1 \quad i = 1, \ldots, N; \quad j = N+1, \ldots, N+M. \quad (9)$$

The explanation of these constraint sets are as follows. Constraints (2) require that every hospital receive a shipment from some vehicle; (3) are the vehicle capacity constraints; (4) are the maximum travel distance constraints (note, it is implicitly assumed that $Q_i \leqslant C_k$ for $i = 1, \ldots, N$ and $k = 1, \ldots, n$); (5) require that graph $\mathcal{L}$ corresponding to x is connected; (6) imply that a vehicle departs from a point h if and only if it enters there (conservation of flow); (7) contains the coupling constraints between variables $x = \{x_{ijk}\}$ and $y = \{y_{ij}\}$. It means that if there is vehicle k passing from both hospital i ($\Sigma_{h=1}^{N+M} x_{ihk} = 1$) and from bank j ($\Sigma_{h=1}^{N+M} x_{jhk} = 1$) then hospital i is assigned to bank j ($y_{ij} \geqslant 1 + 1 - 1 = 1$).

In Problem 1 the variables $x = \{x_{ijk}\}$ correspond to the routing of the periodic delivery vehicles and the variables $y = \{y_{ij}\}$ correspond to the allocations of the hospitals to the blood banks. For a given $x = \{x_{ijk}\}$, $y = \{y_{ij}\}$ is uniquely determined, but the converse is not true; if we are given the allocations, a series of M vehicle dispatch problems have to be solved, in order to obtain the routings. Problem 1 has a finite feasible solution set and a nonempty optimal solution set. However, the underlying Multiple Vehicle Dispatch Problem (MVDP) makes it a complex integer programming problem. For N of any significant size ($N \geqslant 20$), the BTAP is too large to be solved by conventional mathematical programming techniques in a reasonable amount of time. In the following section we will introduce and discuss a good heuristic solution procedure for the BTAP. First, let us thoroughly examine the subproblems of the BTAP that we will be using in the heuristic approaches.

In Problem 1, if γ_i, $i = 1, \ldots, N$ are small or emergency costs negligible (actual γ_i's range from .0002 to .06 when optimal ordering policies are followed, see Pierskalla and Yen [35]) and the function $s(\ell, k)$ is essentially constant, then

$$z^2(x) = \sum_{i=1}^{N+M} \sum_{j=1}^{N+M} \sum_{k=1}^{n} d_{ij}x_{ijk} \tag{10}$$

would be the dominating term in the objective function (1). Then we could just solve the MVDP,

Problem 2

$$\min \sum_{i=1}^{N+M} \sum_{j=1}^{N+M} \sum_{k=1}^{n} d_{ij}x_{ijk} \tag{11}$$

subject to

$$\sum_{k=1}^{n} \sum_{j=1}^{N+M} x_{ijk} = 1 \quad i = 1, \ldots, N \tag{12}$$

$$\sum_{j=1}^{N+M} \sum_{i=1}^{N} Q_i x_{ijk} \leqslant C_k \quad k = 1, \ldots, n \tag{13}$$

$$\sum_{j=1}^{N+M} \sum_{i=1}^{N+M} d_{ij}x_{ijk} \leqslant D_k \quad k = 1, \ldots, n \tag{14}$$

$$\sum_{\{i:H_i \in S\}} \sum_{\{j:H_j \in \bar{S}\}} \sum_{k=1}^{n} x_{ijk} \geqslant 1 \quad \text{for all } (S, \bar{S}) \tag{15}$$

$$\sum_{j=1}^{N+M} x_{hjk} = \sum_{i=1}^{N+M} x_{ihk} \quad k = 1, \ldots, n; h = 1, \ldots, N+M \tag{16}$$

$$x_{ijk} = 0, 1 \qquad i = 1, \ldots, N+M; j = 1, \ldots, N+M; \; k = 1, \ldots, n \tag{17}$$

in order to obtain the optimal x^* for Problem 1. The optimal allocations, y^*, would then be uniquely determined by x^*.

On the other hand, if γ_i, $i = 1, \ldots, N$ are relatively large (which might happen under nonoptimal ordering policies) or system costs and periodic delivery costs are negligible, then

$$z^3(y) = \sum_{i=1}^{N} \sum_{j=N+1}^{N+M} \gamma_i d_{ij} y_{ij}$$

would be the dominating term in the objective function. Then we could just solve the allocation problem,

Problem 3

$$\min \sum_{i=1}^{N} \sum_{j=N+1}^{N+M} \gamma_i d_{ij} y_{ij} \tag{18}$$

subject to

$$\sum_{j=N+1}^{N+M} y_{ij} = 1 \qquad i = 1, \ldots, N \tag{19}$$

$$y_{ij} = 0, 1 \quad i = 1, \ldots, N; \; j = N+1, \ldots, N+M \tag{20}$$

in order to get the optimal y° for Problem 1. Then, optimal routings, x°, would be obtained by solving a vehicle dispatch problem for each one of the M regions determined by y°.

Let x^* be an optimal solution of Problem 2. Let y^* be the allocations determined by x^*. Let y° be an optimal solution of Problem 3. Define $P_h(y)$ to be the set consisting of central bank N+h and the hospitals which it serves. Let $P(y) = \{P_1(y), P_2(y), \ldots, P_m(y)\}$ such that $H_{N+h} \in P_h(y)$, $h = 1, \ldots, M$; $H_i \in P_h(y)$ if and only if $y_{i,N+h} = 1$, $i = 1, \ldots N$; $h = 1, \ldots, M$. (It should be noted that $\cup_{j=1}^{M} P_j(y) = H$ and $P_{j_1}(y) \cap P_{j_2}(y) = \phi$, $j_1 \neq j_2$; $j_1, j_2 = 1, \ldots, M$; for all y feasible in Problem 1.)

Let x° be the routings obtained by solving the following vehicle dispatch problem for $h = 1, \ldots, M$ (and renumbering the vehicles so that each corresponds to a different circuit).

Problem 4

$$\min \sum_{\{i:H_i \in P_h(y^\circ)\}} \sum_{\{j:H_j \in P_h(y^\circ)\}} \sum_{k=1}^{n} d_{ij}x_{ijk} \tag{21}$$

subject to

$$\sum_{k=1}^{n} \sum_{\{j:H_j \in P_h(y^\circ)\}} x_{ijk} = 1 \text{ for all } i \text{ such that } H_i \in P_h(y^\circ) \tag{22}$$

$$\sum_{\{j:H_j \in P_h(y^\circ)\}} \sum_{\{i:H_i \in P_h(y^\circ)\}} Q_i x_{ijk} \leqslant C_k \quad k = 1, \ldots, n \tag{23}$$

$$\sum_{\{j:H_j \in P_h(y^\circ)\}} \sum_{\{i:H_i \in P_h(y^\circ)\}} d_{ij}x_{ijk} \leqslant D_k \quad k = 1, \ldots, n \tag{24}$$

$$\sum_{\{i:H_i \in S\}} \sum_{\{j:H_j \in \bar{S}\}} \sum_{k=1}^{n} x_{ijk} \geq 1 \quad \text{for all } (S, \bar{S}) \tag{25}$$

where S is any proper subset of $P_h(y^\circ)$ and $\bar{S}$ its complement

$$\sum_{\{j:H_j \in P_h(y^\circ)\}} x_{\ell jk} = \sum_{\{i:H_i \in P_h(y^\circ)\}} x_{i\ell k} \quad \text{for all } \ell \text{ such that} \tag{26}$$

$H_\ell \in P_h(y^\circ)$ and $k = 1, \ldots, n$

$$x_{ijk} = 0, 1 \ \{i:H_i \in P_h(y^\circ)\}, \{j:H_j \in P_h(y^\circ)\}, k = 1, \ldots, n \quad (x_{iik} = 0). \tag{27}$$

It directly follows from the above definitions that

$$z^2(x^*) \leq z^2(x^\circ)$$

$$z^3(y^\circ) \leq z^3(y^*)$$

and if the systems costs are essentially constant

$$z^2(x^*) + z^3(y^\circ)$$

would be a good lower bound on the optimal value of Problem 1.

Heuristic Solution Procedures for the BTAP

Let $EX(j_1, j_2)$ be the set of all hospitals such that $H_i \in EX(j_1, j_2)$ if and only if $H_i \notin S$ and $H_i \in P_{j_1}(y^*)$, $H_i \in P_{j_2}(y^\circ)$. In other words $H_i \in EX(j_1, j_2)$ means that hospital H_i would have been allocated to bank H_{N+j_1} if only periodic delivery costs were minimized, and hospital H_i would have allocated to bank H_{N+j_2}, if only emergency delivery costs were minimized.

The basic idea behind the solution procedures that will be discussed in this section is fairly simple. First, the feasible solutions (x^*, y^*) and (x°, y°) of Problem 1, discussed in the previous section, are obtained, Then, they are compared and for each pair $(j_1, j_2), j_1 \neq j_2; j_1, j_2 = 1, \ldots, M$, hospitals $H_i \in EX(j_1, j_2)$ are removed temporarily from $P_{j_1}(y^*)$ and inserted into $P_{j_2}(y^*)$. [Let us call this operation an exchange between sets $P_{j_1}(y^*)$ and $P_{j_2}(y^*)$]. After each exchange, Problem 4 is solved for the two sets $P_{j_1}(y^*)$ and $P_{j_2}(y^*)$, under consideration, in order to obtain the corresponding components of the variable (x, y) of Problem 1. The components of (x, y) corresponding to sets $P_j(y^*)$, $j \neq j_1, j \neq j_2$, are left unchanged. The resulting feasible solution, to Problem 1, is compared with the better of (x^*, y^*) and (x°, y°). If there is a decrease in the objective function value, $z^*(x, y)$, the exchange is made permanent. Then another exchange is considered and so on.

Two algorithms have been developed, both based on the idea described above, and differing only in the way the exchanges implied by the sets "$EX(j_1, j_2), j_1 \neq j_2, j_1, j_2 = 1, \ldots, M$" are ordered and executed. The first algorithm is reasonably fast, but the set of exchanges it tests is not very large; it tests the elements in $\cup_{(j_1,j_2)} EX(j_1, j_2)$ only one by one and independent of each other. The second algorithm tests a larger set, besides testing the elements in $\cup_{(j_1, j_2)}$, $EX(j_1, j_2)$, it also tests many different combinations of them. Hence, the second algorithm produces a better solution. Unfortunately, these latter tests are time consuming, and they slow down the algorithm considerably. Both of these algorithms were originally developed assuming no vehicle capacity constraints and no maximum number of stops per vehicle type constraints. In other words, constraint sets (3) and (4) of Problem 1 were assumed to be nonbinding [in which case, of course, the underlying MVDP reduces to the Mutliple Traveling Salesman Problem (MTSP)]. Then both algorithms were extended to provide solutions to the BTAP also when the constraints on the vehicles were binding. It should be noted that, in these algorithms heuristic procedures are employed to solve Problems 2 and 4. So, (x^*, y^*) obtained is not the optimal solution of Problem 2, but a near optimal solution. Similarly, x° is a combination of near optimal solutions obtained by solving Problem 4 for $h = 1, \ldots, M$. Therefore, the inequality

$$z^2(x^*) \leq z^2(x) \quad \text{for all } x \text{ feasible for Problem 2}$$

which is always true when x^* is optimal, might not always hold in our heuristic approaches, since there x^* is only "near optimal."

Algorithm 1

The following algorithm is a solution procedure for the BTAP, when the constraints on the vehicles [constraint sets (3) and (4) of Problem 1] are not binding. In this algorithm the sets $EX(j_1, j_2)$ are ordered such that for all $H_{i_1}, H_{i_1+1} \in EX(j_1, j_2)$

$$d_{i_1 j_1} - d_{i_1 j_2} \leq d_{i_1+1, j_1} - d_{i_1+1, j_2}.$$

In other words the marginal decrease in the emergency delivery cost if H_{i_1} were to be placed in $P_{j_2}(y^*)$ is greater than the decrease if H_{i_1+1} were to be placed in $P_{j_2}(y^*)$. Then, each set $EX(j_1, j_2), j_1 \neq j_2, j_1, j_2 = 1, \ldots, M$ is considered one at a time. Starting with the first element, all elements of the set $EX(j_1, j_2)$, under consideration, are, one by one temporarily removed from $P_{j_1}(y^*)$ and inserted into $P_{j_2}(y^*)$. Then two traveling-salesman problems are solved for these two sets. The resulting feasible solution to Problem 1 is compared with (x^*, y^*) and (x°, y°); if it is better, the change is made permanent.

Assume the following parameters are given: the coordinates of N hospitals and M banks; the function $s(\ell, k)$; the daily blood usage, Q_i, in each hospital H_i; the period for the periodic deliveries; and the expected number of emergency deliveries γ_i for each hospital H_i in one period. Then the steps of algorithm 1 are as follows:

1: Read in all the Data.

2: Compute the distance matrix, $D = \{d_{ij}\}$, $i = 1, \ldots, N{+}M$; $j = 1, \ldots, N{+}M$.

3: For each hospital H_{i_1}, determine the closest bank H_{N+j_1}. Set $y^\circ_{i_1 j_1} = 1$. (This step will determine y°.)

4: For each set $P_j(y^\circ)$, $j = 1, \ldots, M$, solve the traveling salesman problem, using the convex hull algorithm or any traveling salesman algorithm available. The convex hull algorithm is given in Or [33] and in Or and Pierskalla [34]. (This step will determine x°.)

5: Apply the multiple assignment algorithm or any multiple traveling salesman algorithm available to the given set of points. The multiple assignment algorithm is also given in the two references in step 4 above. (This step will determine y^*.)

6: For each set $P_j(y^*)$, $j = 1, \ldots, M$, solve the traveling salesman problem, using the convex hull algorithm. (This step will determine x^*.)

7: Let $z_{min} = \min[z(x^*, y^*), z(x^\circ, y^\circ)]$, where

$$z(x, y) = \sum_{i=1}^{N+M} \sum_{j=1}^{N+M} d_{ij}x_{ij} + \sum_{i=1}^{N} \sum_{j=N+1}^{N+M} \gamma_i d_{ij} y_{ij}$$

$$+ \sum_{j=N+1}^{N+M} s\left(\sum_{i=1}^{N} y_{ij}, 300 \sum_{i=1}^{N} Q_i y_{ij} \right).$$

Note that $z(x, y)$ is just the objective function of Problem 1 with the subscript k of x and summation over k deleted. Since there is only a single vehicle serving each supply point, this deletion does not affect the optimal value of Problem 1.

Let $(\tilde{x}, \tilde{y}) = (x^*, y^*)$

8: a) Execute the following operations for each pair (j_1, j_2), $j_1 \neq j_2, j_1, j_2 = 1, \ldots, M$. When done, go to 9.

b) Determine the set $EX(j_1, j_2)$, if it is empty go to 8a.

c) Order $EX(j_1, j_2)$ such that for all $H_{i_1}, H_{i_1+1} \in EX(j_1, j_2)$ $d_{i_1 j_1} - d_{i_1 j_2} \leqslant d_{i_1+1, j_1} - d_{i_1+1, j_2}$.

d) Go to 8a.

9: a) Execute the following operations for each pair (j_1, j_2), $j_1 \neq j_2, j_1, j_2 = 1, \ldots, M$. When done, go to 10.

b) If $EX(j_1, j_2)$ is empty, go to 9a.

c) Remove the first element of $EX(j_1, j_2)$; let it be H_{i_1}. Define $\bar{y} = \{\bar{y}_{ij}\}$ such that $\bar{y}_{i_1 j_1} = 0$, $\bar{y}_{i_1 j_2} = 1$, and $\bar{y}_{ij} = \tilde{y}_{ij}$ otherwise.

d) Solve the traveling salesman problem for the sets $P_{j_1}(\bar{y})$ and $P_{j_2}(\bar{y})$. Store the resulting tours in $\bar{x} = \{\bar{x}_{ij}\}$ such that

$\bar{x}_{ij} = 1$ if $H_i \in P_{j_1}(\bar{y}), H_j \in P_{j_1}(\bar{y})$ and the convex hull algorithm solution to $P_{j_1}(\bar{y})$ contains an edge between H_i and H_j.

$\bar{x}_{ij} = 0$ if $H_i \in P_{j_1}(\bar{y}), H_j \in P_{j_1}(\bar{y})$ and the convex hull solution to $P_{j_1}(\bar{y})$ does not contain an edge between H_i and H_j.

$\bar{x}_{ij} = 1$ if $H_i \in P_{j_2}(\bar{y}), H_j \in P_{j_2}(\bar{y})$ and the convex hull solution to $P_{j_2}(\bar{y})$ contains an edge between H_i and H_j

$\bar{x}_{ij} = 0$ if $H_i \in P_{j_2}(\bar{y}), H_j \in P_{j_2}(\bar{y})$ and the convex hull solution to P_{j_2} does not contain an edge between H_i and H_j.

$\bar{x}_{ij} = \tilde{x}_{ij}$ otherwise.

e) Compare $z_{\min}$ with $z(\bar{x}, \bar{y})$. If $z_{\min}$ is smaller go to 9b; otherwise proceed.

f) Update $\tilde{x}, \tilde{y}$, and $z_{\min}$. $\tilde{x} = \bar{x}, \tilde{y} = \bar{y}$, and $z_{\min} = z(\bar{x}, \bar{y})$.

g) Go to 9b.

10: All of the exchanges are completed, terminate. $(\tilde{x}, \tilde{y})$ is the resulting near optimal solution; $z_{\min}$ is the resulting near optimal value.

In most applications, the above algorithm will produce a very good solution in a reasonable amount of time (see the next section for the actual execution times). However, in some extreme cases (i.e., cases in which most of the sets $EX(j_1, j_2)$ are very large), it could be very time consuming, considering that two traveling salesman problems are solved after each exchange and there are as many exchanges as the total number of elements in the sets $EX(j_1, j_2), j_1 \neq j_2; j_1, j_2 = 1, \ldots, M$ (i.e., there are "$\sum_{j_1=1}^{M} \sum_{j_2=1, j_2 \neq j_1}^{M} |EX(j_1, j_2)|$" exchanges

Algorithm 2

The following algorithm is a solution procedure for the BTAP, when the constraints on the vehicles are not binding. In this algorithm, the sets $EX(j_1, j_2)$ are ordered such that for all $H_i, H_{i+1} \in EX(j_1, j_2)$

$$d_{i_1 j_1} - d_{i_1 j_2} \leqslant d_{i_1+1, j_1} - d_{i_1+1, j_2}.$$

Then each pair of sets $EX(j_1, j_2), EX(j_2, j_1), 1 \leqslant j_1 < j_2 \leqslant M$ are considered one at a time. The first elements H_{i_1}, H_{i_2} of the sets under consideration, $EX(j_1, j_2)$ and $EX(j_2, j_1)$, respectively, are removed, and the following temporary exchanges are done in the given order:

H_{i_1} is removed from $P_{j_1}(y^*)$ and inserted into $P_{j_2}(y^*)$.

H_{i_1} and one of the hospitals adjacent to it [adjacent in the graph defined by $(\tilde{x})$] are removed from $P_{j_1}(y^*)$ and inserted into $P_{j_2}(y^*)$.

H_{i_1} and the other hospital adjacent to it [adjacent in the graph defined by $(\tilde{x})$] are removed from $P_{j_1}(y^*)$ and inserted into $P_{j_2}(y^*)$.

H_{i_1} and both of the hospitals adjacent to it are removed from $P_{j_1}(y^*)$ and inserted into $P_{j_2}(y^*)$.

H_{i_2} is removed from $P_{j_2}(y^*)$ and inserted into $P_{j_1}(y^*)$.

H_{i_2} and one of the hospitals adjacent to it are removed from $P_{j_2}(y^*)$ and inserted into $P_{j_1}(y^*)$.

H_{i_2} and the other hospital adjacent to it are removed from $P_{j_2}(y^*)$ and inserted into $P_{j_1}(y^*)$.

H_{i_2} and both of the hospitals adjacent to it are removed from $P_{j_2}(y^*)$ and inserted into $P_{j_1}(y^*)$.

After each exchange the new feasible solution to Problem 1 is determined (as described in the previous section) and compared with (x°, y°), (x^*, y^*). If it is a better solution, the corresponding exchange is made permanent. This process is continued until both of the sets $EX(j_1, j_2)$, $EX(j_2, j_1)$, for j_1, j_2 under consideration, are empty. (Details of this algorithm are given in Or [33].

Extensions of Algorithm 1 and Algorithm 2

The algorithms presented in the previous section may be extended quite easily, to provide solutions also in the cases in which the constraints on the vehicles are binding (i.e., the underlying multidepot problem is the MVDP rather than the MTSP). One possible extension would be to use the Gillett and Miller sweep algorithm instead of the convex hull algorithm in steps 4, 6, 9d of algorithm 1 and in algorithm 2. Let us call this extension 1. Another extension would be to leave the first nine steps of the algorithm the same and change step 10 to:

10: Consider $\tilde{y}$ to be the near optimal allocations. For each set $P_j(\tilde{y}), j = 1, \ldots, M$, solve the vehicle dispatch problem using the sweep algorithm.

Let us call this last extension, extension 2. Notice that in extension 2 the sweep algorithm is applied only M times (in step 10), whereas in extension 1, it is applied M times in step 4, M times in step 6 and then twice after each exchange (it replaces the convex hull algorithm). Hence, as the sweep algorithm is slower than the convex hull algorithm, extension 1 is slower than extension 2.

Considering how large and complex the BTAP is, algorithms 1 and 2 and their extensions produce some very acceptable feasible solutions and give us some insights about the BTAP, as will be discussed in the next section. However, because of their size and complexity we should note that we do not really know how good (or bad) a solution we are getting from these algorithms. For example, we do not know whether the allocations, $\tilde{y}$ (which were obtained for the MTSP and used in step 10 of extension 2, in solving the MVDP) are near optimal allocations for the MVDP or not. Similarly, we do not know how good the assignment for the MVDP will be since it also was developed for the MTSP. Also, the solutions from the sweep algorithm are not guaranteed to be near optimal. These are all relevant shortcomings of our solution procedures and more research will be done to resolve them. (These algorithms and their extensions are available in Or and Pierskalla [34]).

Computational Results

In this section we mention some of the results for the BTAP using the actual data for all hospitals in the Greater Metropolitan Chicago area. Fifteen cases were run to obtain the solutions (x^*, y^*), (x°, y°), $(\tilde{x}, \tilde{y})$, for different choices of bank locations and numbers. Algorithm 1, algorithm 2 and extension 2 were applied and their performances compared.

Average Execution Times

Solution Obtained	*Time in Seconds on CDC 6400*
(x°, y°)	3.6
(x^*, y^*)	9.5
$(\tilde{x}, \tilde{y})$ using algorithm 1	41.9
$(\tilde{x}, \tilde{y})$ using algorithm 2	67.8
MVDP using extension 2 (given y°, y^* or $\tilde{y}$)	3.2

Based on these cases, we can make a few interesting observations.

1. Algorithm 2, which is far slower than algorithm 1, produces very little significant improvement in the solution.

2. The problem is insensitive to the parameter γ_i, $i = 1, \ldots, N$. (Even for a large value for γ_i, $i = 1, \ldots, N$, the periodic delivery cost considerations always dominated the emergency cost considerations.)

3. In the existence of binding capacity constraints, $\tilde{y}$ produces very little significant improvement over y°.

These observations, provided that they hold in other cases in future research, imply that complex heuristics do very little over simple heuristics to improve the solution of the BTAP. Consequently, if the first observation is true in general, algorithm 2 may be abandoned with considerable savings in execution time. If the last observation is true in general, one might simply use the allocations given by y°, which are very easy to determine, in the solution procedure of the BTAP, and avoid all the complicated processes to obtain y^* or $\tilde{y}$. The second observation reduces the need for accurate estimates for γ_i, the expected number of emergency deliveries to hospital H_i in one period. These observations, of course, require further testing. This effort is left for future research and data gathering in other regions of the country.

As examples, the results of two of the fifteen cases are presented in Figs. 1 and 2 and Tables 1 and 2 respectively. These examples are representative of the other cases. In each example hospitals 1, 2 and 3 (Northwestern Memorial, Michael Reese, and Rush-Presbyterian-St. Luke, respectively)

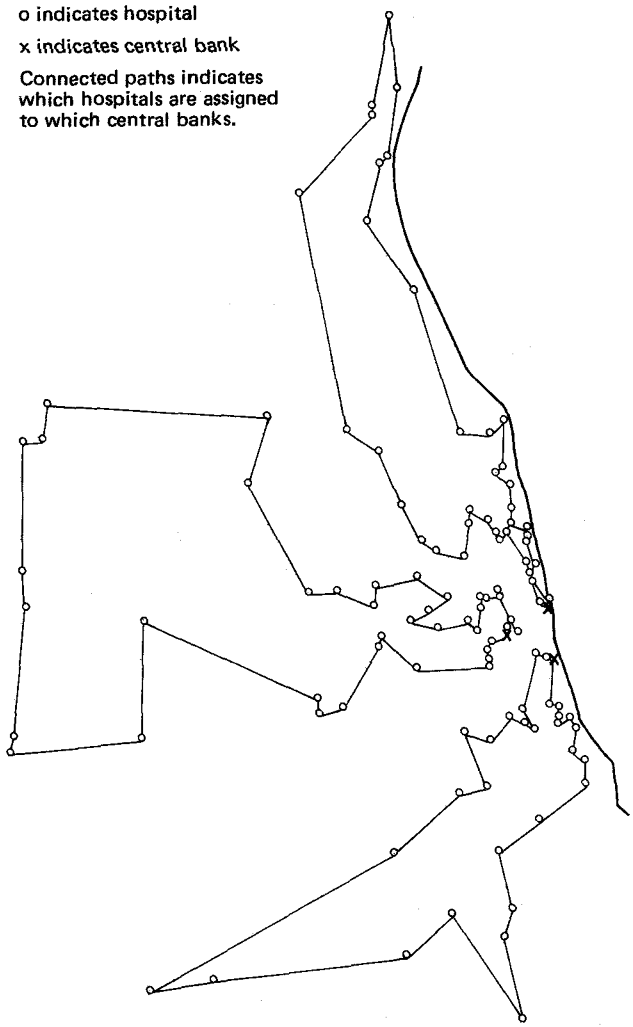

Fig. 1. Allocation based on emergency, periodic, and system costs metropolitan Chicago inter-hospital blood transportation network.

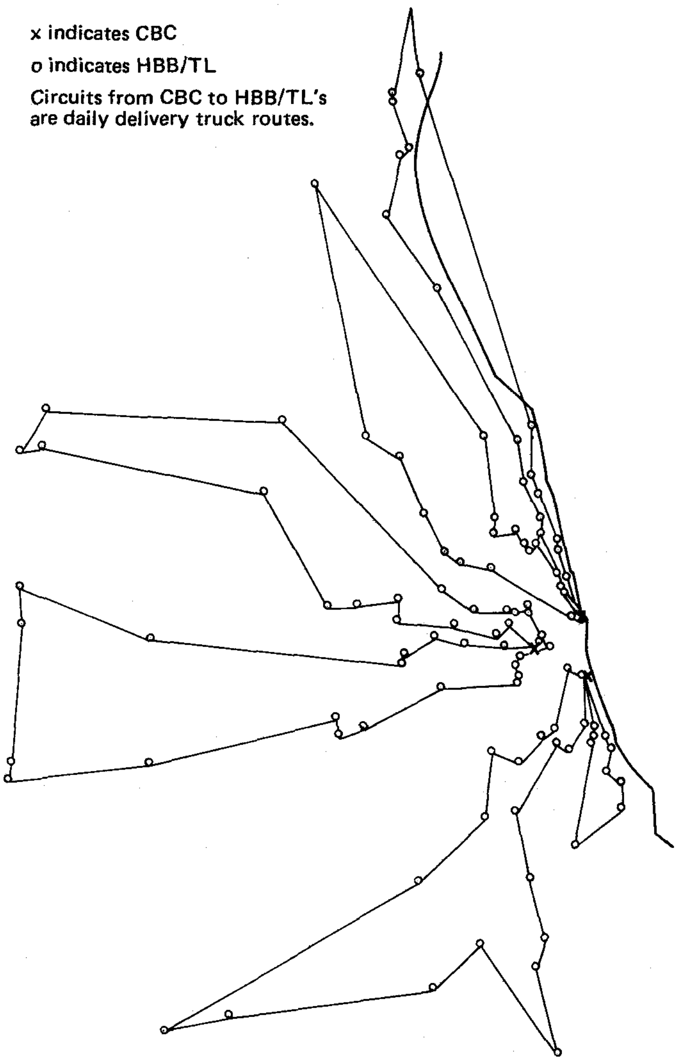

Fig. 2. Allocation based on emergency routing system costs metropolitan Chicago inter-hospital blood transportation network.

were chosen for the central banks. This is because the decision maker had a preference, external to the BTAP, for the banks to be located at those hospitals. All other cases and results may be found in Or [33].

Table 1: Identification of hospitals assigned to central bank and the annual volume at each central bank shown in Fig. 1.

Bank 1, Identification-Hospital 1	
Number of hospitals in the system	42
Amount of blood used in the system	54178
Bank 2, Identification-Hospital 2	
Number of hospitals in the system	30
Amount of blood used in the system	53524
Bank 3, Identification-Hospital 3	
Number of hospitals in the system	45
Amount of blood used in the system	84502

Table 2: Identification of hospitals and number of units delivered daily on each route and the annual volume at each central bank shown in Fig. 2.

Bank 1, Routing	Identification-Hospital 1			
Truck No. 1	Number of Stops:	20	Number of Units:	96
Truck No. 2	Number of Stops:	20	Number of Units:	99
Truck No. 3	Number of Stops:	1	Number of Units:	12
Amount of blood used in the system: 54178				
Bank 2, Routing	Identification-Hospital 2			
Truck No. 1	Number of Stops:	20	Number of Units:	132
Truck No. 2	Number of Stops:	9	Number of Units:	63
Amount of blood used in the system: 53524				
Bank 3, Routing	Identification-Hospital 3			
Truck No. 1	Number of Stops:	20	Number of Units:	107
Truck No. 2	Number of Stops:	20	Number of Units:	133
Truck No. 3	Number of Stops:	4	Number of Units:	66
Amount of blood used in the system: 84502				

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